Efficient worth-sharing in dynamic coalitions and behavioral coalition structure generation

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1 outline

coalition structures and optimality

artificial intelligence (AI) and search

game theory and stable optimal coalition structures

behavioral generation: payoff comparisons

coalitions behavior: efficient worth-sharing

two alternative sharing rules

agents behavior: random comparisons

conclusion

2 notation A

$N = \{1, \ldots, n\}$ MAS population

$2^N = \{A : A \subseteq N\}$ power set of coalitions

$P = \left\{A_1, \ldots, A_{|P|}\right\} \subset 2^N \setminus \{\emptyset\}$ coalition structure:

$\bigcup_{1 \leq k \leq |P|} A_k = N$ and $A_h \cap A_k = \emptyset$ for $1 \leq h < k \leq |P|$

$A \in P$ is a block of coalition structure (or partition) P

$v : 2^N \rightarrow \mathbb{R}$ coalitional game: $v(A) =$ worth of A

$\sum_{A \in P} v(A) =$ worth of coalition structure P

optimal coalition structures have maximum worth

$\mu^v : 2^N \rightarrow \mathbb{R}$ is the Möbius inversion (derivative) of v

$\mu^v(A) = v(A) - \sum_{B \subset A} \mu^v(B)$ for every $A \in 2^N$

3 notation B

$v(\emptyset) = 0$ and $N \supseteq A \supseteq B \Rightarrow v(A) \geq v(B)$ (monotone)

if v is superadditive (i.e., for every $A, B \in 2^N$, $A \cap B = \emptyset$

$$\begin{aligned} v(A \cup B) &\geq v(A) + v(B)), \\ \text{then } v(N) &\geq \sum_{A \in P} v(A) \end{aligned}$$

for every P; the coarsest coalition structure is optimal

if v is subadditive (i.e., for every $A, B \in 2^N$, $A \cap B = \emptyset$

$$\begin{aligned} v(A \cup B) &\leq v(A) + v(B)), \\ \text{then } \sum_{i \in N} v(i) &\geq \sum_{A \in P} v(A) \end{aligned}$$

for every P; the finest coalition structure is optimal.

if merging is neither always better, nor always worse, then optimal coalition structures are generic; if $\mu^v(A) \geq 0$ for every $A \in 2^N$, then $v(N) > \sum_{A \in P} v(A)$ for every P

4 focus

finding optimal coalition structures is NP-hard

AI approach: (branch-and-bound) search, non-behavioral

coalition structure generation defines a (possibly stochastic) time pattern $P^0, P^1, \ldots, P^T$ of coalition structures

blocks $A = A^t \in P^t$ of prevailing coalition structures $P^t, t \geq 0$ are dynamic coalitions, changing in time t

behavioral coalition structure generation: selfish behavior of computationally bounded agents determines evolution

this may approach the problem of maximizing the (expected) stream of worth of prevailing coalition structures with an underlying game v **changing over time**

5 a coalition formation game

given coalitional game v, every agent $i \in N$ has strategy set $S_i = \left\{A \in 2^N : A \ni i\right\}$

n-tuple $s = (s_1, \ldots, s_n) \in S = \underset{1 \le i \le n}{\times} S_i$ of strategies results in coalition structure P_s where for $i, j \in N$

$$s_i = s_j \Leftrightarrow \{i, j\} \subseteq A \in P_s$$

agent $i \in N$ has utility $u_i : S \rightarrow \mathbb{R}$ such that for every $s \in S$, if $i \in A \in P_s$, then

$$u_i(s) = \sum_{A \supseteq B \ni i} \frac{\mu^v(B)}{|B|}$$

this strategic game is a potential game (i.e., it admits a potential, see Monderer-Shapley (1996)), and thus has at least one (pure-strategy Nash) equilibrium $s^* \in S$

equilibria s^* result in optimal coalition structures P_{s^*}

6 payoff comparisons

behavioral coalition structure generation cannot be strategic, in a strict sense, because the population is large

at any time, each agent is a member of some (dynamic) coalition and receives a share of its worth

periodically, agents compare their payoff (or a weighted average) with that of a randomly selected other agent; if the former weakly exceeds the latter, then they stay; otherwise, they move into the other agent's coalition

coalitions' payoff policy becomes crucial for performance; it determines the only little information agents have when choosing whether to move or stay

from coalition formation games, worth-sharing through Möbius inversion should be superior for performance

7 alternative payoffs

main idea: comparing two efficient sharing rules; basically, for every $i \in A \in P^t, t \geq 0$

$$\begin{aligned}
\phi_i\left(v^A\right) &= \frac{v(A)}{|A|} \\
\phi_i^*\left(v^A\right) &= \sum_{B \subseteq A : B \ni i} \frac{\mu^v(B)}{|B|}
\end{aligned}$$

example: $v(A) = \gamma(|A|), \gamma : \{0, 1, \ldots, n\} \to \mathbb{R}$

$$v(A) = \frac{|A|}{1 + (|A| - m)^2} \quad \text{(symmetric)}$$

$n = 60$; draw $m \in \{2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30\}$

$$\max_P \sum_{A \in P} v(A) = \frac{n}{m} \cdot \frac{m}{1 + (m - m)^2} = n \text{ for every } m$$

v symmetric $\Rightarrow \phi_i^*\left(v^A\right) = \phi_i\left(v^A\right), i \in A \in 2^N$

yet, introducing memory yields $\phi_i^*\left(v^A\right) \neq \phi_i\left(v^A\right)$

8 membership age (H = memory)

$h(i)$ = age of $i \in A \in P^t, 1 \leq h(i) \leq H \geq 1$

a_h = number of A-members aged $h, a_h \leq |A|$

b_h = number of B-members aged $h, A \supseteq B \ni i$

$$\phi_i(v^A) = \frac{v(A) \cdot h(i)}{a_{h(i)} \cdot \sum_{\substack{h \in H \\ a_h > 0}} h}$$

$$\phi_i^*(v^A) = \sum_{\substack{B \subseteq A \\ B \ni i}} \frac{\mu^v(B) \cdot h(i)}{b_{h(i)} \cdot \sum_{\substack{h \in H \\ b_h > 0}} h} \text{ if } \mu^v(B) \geq 0$$

$$\phi_i^*(v^A) = \sum_{\substack{B \subseteq A \\ B \ni i}} \frac{\mu^v(B) \cdot (H - h(i) + 1)}{b_{h(i)} \cdot \sum_{\substack{h \in H \\ b_h > 0}} h} \text{ if } \mu^v(B) < 0$$

9 agents behavior A

at t, with $P^{t-1} = \left\{A_1, \ldots, A_{|P^{t-1}|}\right\}$ from before,

FIRSTLY a fraction of agents decides whether move,

SECONDLY (new) dynamic coalitions compute payoffs

for each i who decides whether to move do:

with probability ϵ draw $k \in \left\{0, 1, \ldots, |P^{t-1}|\right\}$

if $k = 0$, then i mutates (into a 1-block $\{i\} \in P^t$)

if $i \in A_k$ then i does not move

otherwise i moves into block A_k

with probability $1 - \epsilon$ randomly select some agent j, possibly of same age as i but in another block, and move i into j's block if j does strictly better

10 agents behavior B

ϵ quantifies the random part of the generation process

$1 - \epsilon$ quantifies the drift

rather than keeping these parts fixed, they can change depending on recent performance:

$\epsilon(t) = \epsilon$ if $f(P^{t-1}) - f(P^{t-2}) \geq 0$

$$\epsilon(t) = \frac{f(P^{t-1}) - f(P^{t-2})}{\min\{f(P^{t'}) - f(P^{t'-1}) : 0 < t' \leq t-1\}}$$

if $f(P^{t-1}) - f(P^{t-2}) < 0$

this may be generalized by considering a weighted average of performance over last H cycles

similarly, agents may compare a weighted average of last H received payoffs

11 Conclusion

if this was to be tested through simulations, then:

$v, \mu^v : 2^N \rightarrow \mathbb{R}$ are $2^n - 1$-dimensional (a lot of data!)

v symmetric $\Rightarrow v, \mu^v$ are $n - 1$-dimensional (better)

type-symmetry: N is partitioned into $K \geq 1$ types, with n_k agents of each type $k = 1, \ldots, K$; then, v, μ^v are $(\prod_{1 \leq k \leq K} n_k + 1) - 1$-dimensional

does this fit PEERSIM?

- 'philosophically'

- computationally