# Cooperation's Sensitivity to Network Structure:

## The Case of Individual Learning

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# Overview

_ Interpretation of Cooperation

_ Conflicting results (across fields)

- Context-Preservation in Evolutionary IPD games
- Opinion Dynamics on Networks

_ The Proposed Model

- Results

_ Summary

# Interpretation of Cooperation

# Cooperation, as in the IPD game (A 'strategic' interpretation)

Player 2

| | | C | D |
|---|---|---|---|
| Player 1 | C | 3, 3 | 0, 5 |
| | D | 5, 0 | 1, 1 |

(The Prisoner's Dilemma)

# Context-Preservation in Evolutionary IPD games

# The Iterated Prisoner’s Dilemma

- Repeated Encounters
  - Memory
  - Axelrod’s Tournament x 2
- Strategies
  - “ALLD” is very strong.
  - “TFT” is surprisingly strong.
- Theoretical Results
  - Finite versus Infinite Series
  - TAG-based systems, etc.

# The Evolution of Cooperation

- Cohen-Axelrod-Riolo (CAR):
  - “The Role of Social Structure in the Maintenance of Cooperative Regimes“
- How can cooperation (~trust) evolve spontaneously in a population of selfish agents?
  - I.e., in the IPD framework?

# Evolution of Cooperation #2

- Memory length=1, 4 strategies studied:
  - ALLC (C, C, C)
  - TFT (C, C, D)
  - ATFT (D, D, C)
  - ALLD (D, D, D)

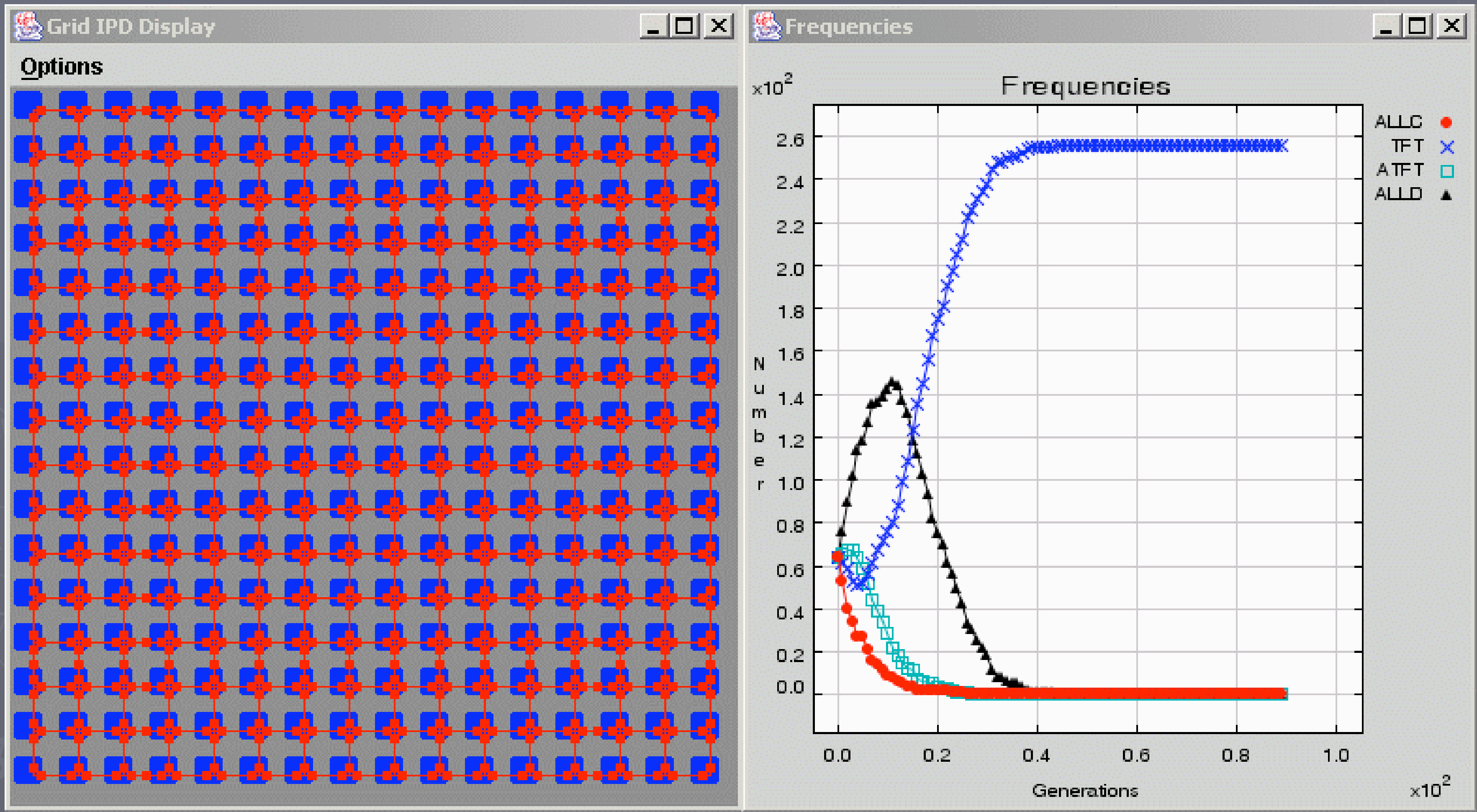
- Agents are assigned a (uniform) random initial strategy.

# Evolution of Cooperation #3

- Various (interaction) network topologies explored ~ average degree of *k*
  - In each round, each agent plays with each of its neighbors.
  - A 4-shots IPD game for each link.

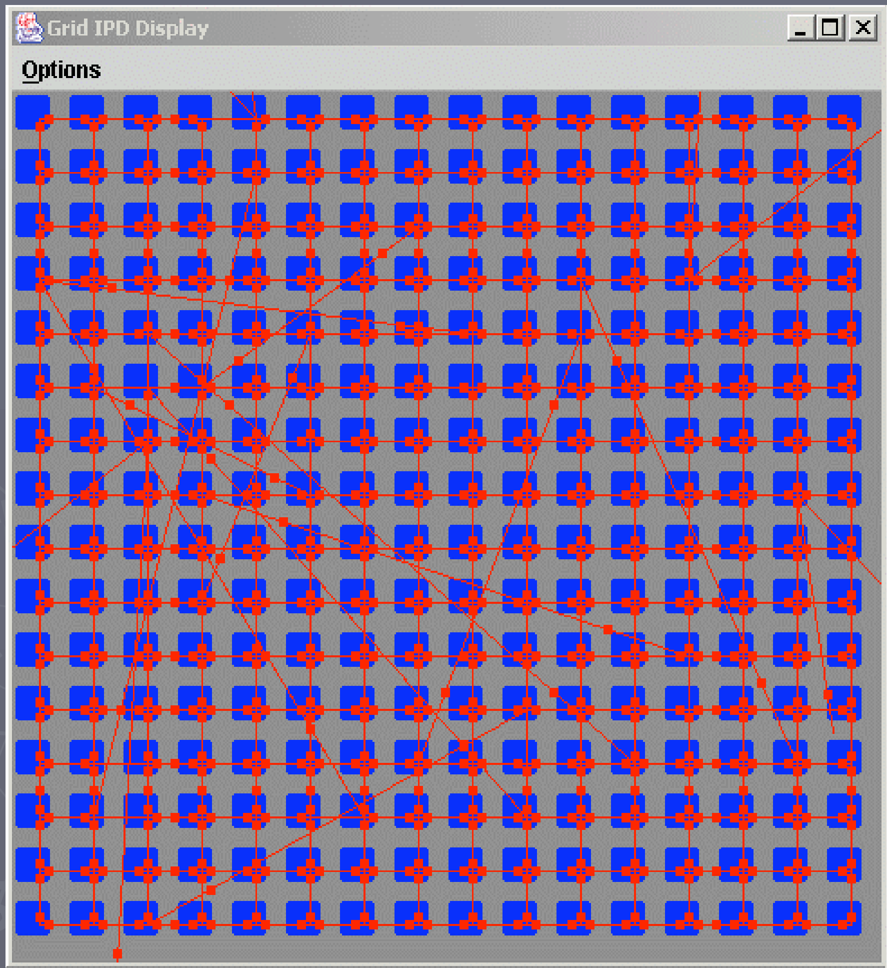
- Evolutionary adaptation:
  - At the end of the round, agents copy the strategy of their most successful neighbors.

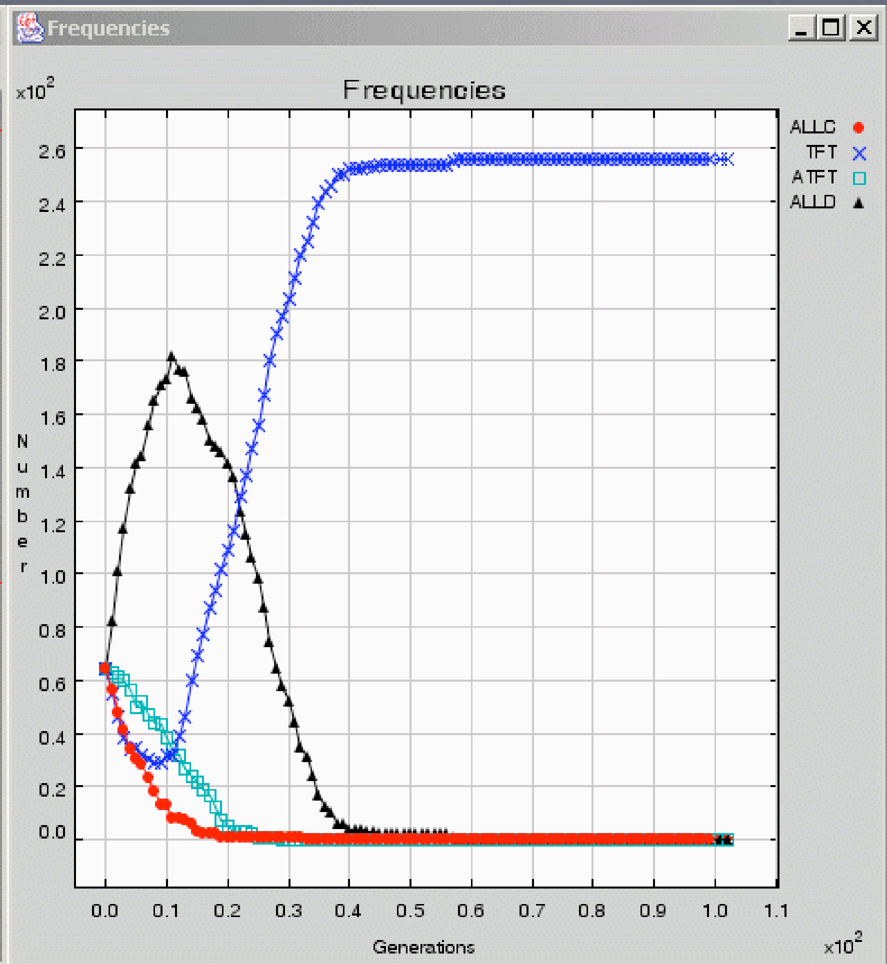
# Evolution of Cooperation #4




On a 2-dimensional torus

# Evolution of Cooperation #5






On a 2-dimensional WS-graph (see later)

# The Essence of the CAR-results

- Cooperation can indeed emerge spontaneously
- Context-preservation is key:
  - The exact structure is not important.
  - Stability is what matters.
    - Dynamic versus Static networks.
  - Depends on the particular values for T, S, P, R ~ *k*.

(The CAR-results apply to a wider set of strategies than discussed here.)

(Works by others show that the *heterogeneity in the degree distribution* plays a significant role as well.)

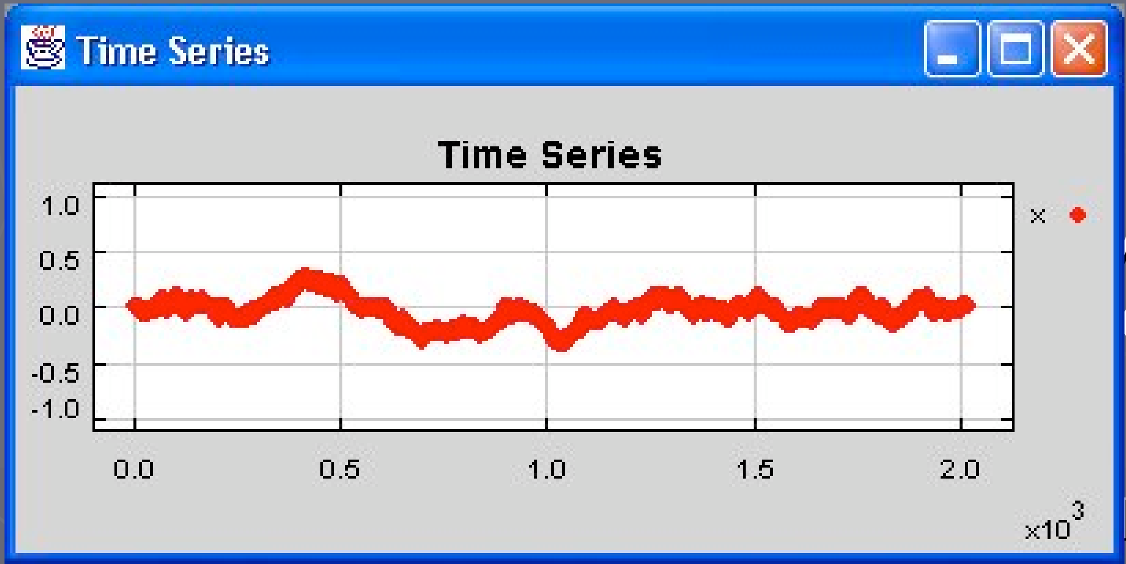
# Opinion Dynamics on Networks

# Discrete Choices on Networks

- An Ising-type model
  - Rooted in Discrete Choice Theory
    (*de facto* standard in econometrics)

- Agents make repeated individual choices from a discrete set C. (|C|=2)
  - Heterogeneous properties:
    - Individual biases can be/are typically taken into account
    - In lack of these, two regimes based on a 'certainty' parameter

# Discrete Choices on Networks

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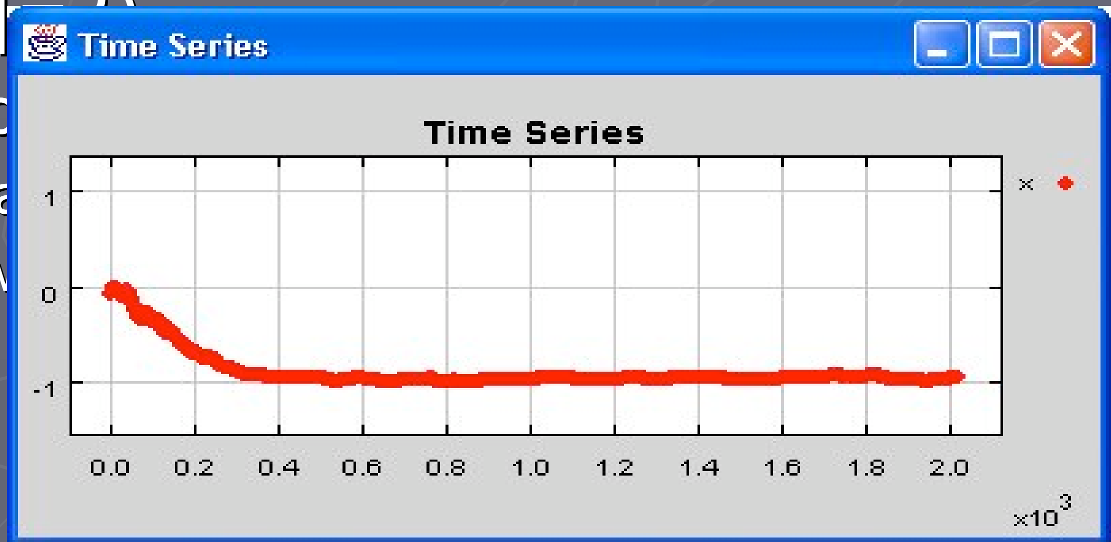


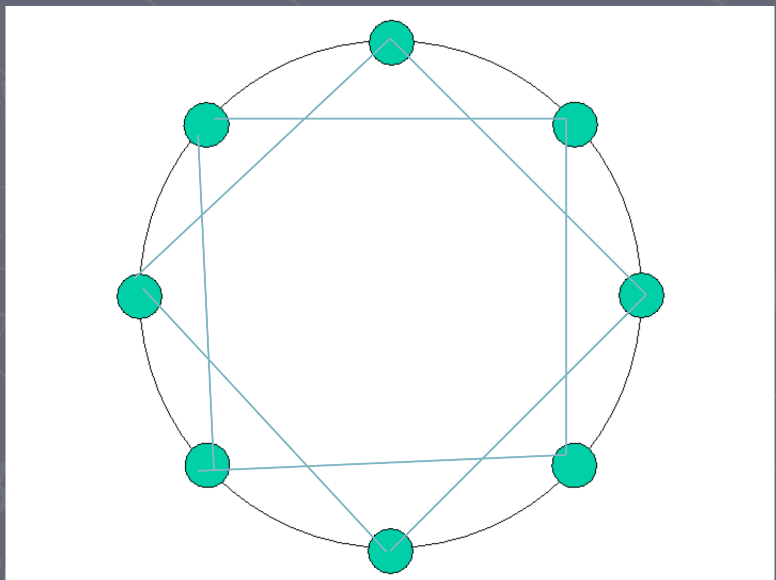
# Discrete Choices on Networks #2

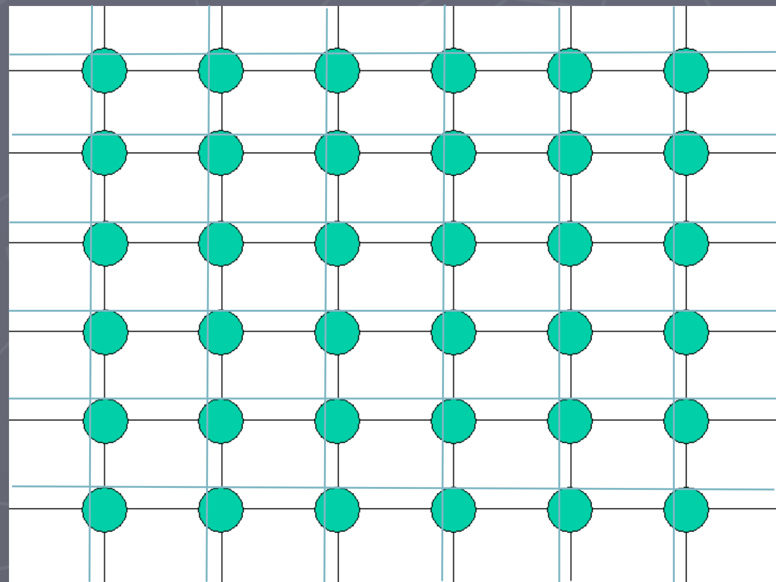
- Adding social influence
  - Mean-field approach (Aoki, Brock & Durlauf)
  - Localized interactions (Dugundji & Gulyas)

- Localized interactions define a network:
  - Special individual-level biases, based on previous decisions of neighbors (in a network).
  - The system-level, aggregate outcome is <u>sensitive to the network structure</u>!

# Watts-Strogatz Networks

- Low average path-length (‘*a small-world…*’)
- High level of clustering
  (‘*a friend of a friend is a friend…*’)





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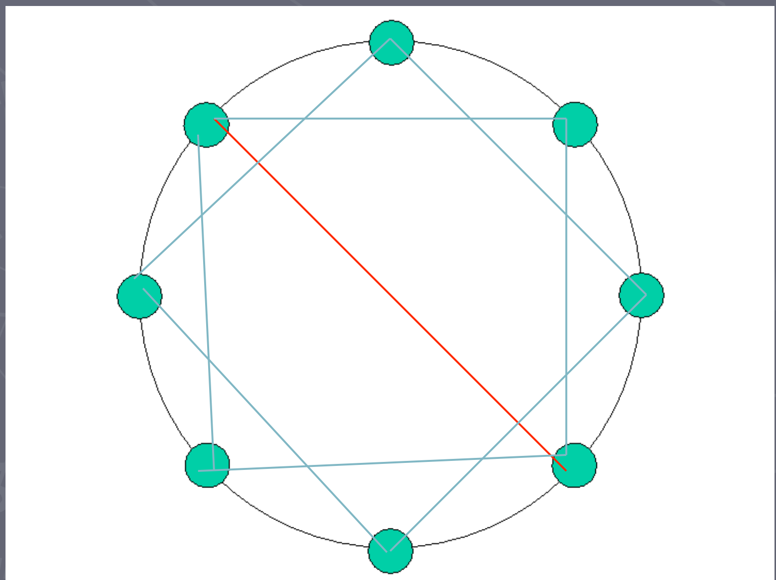
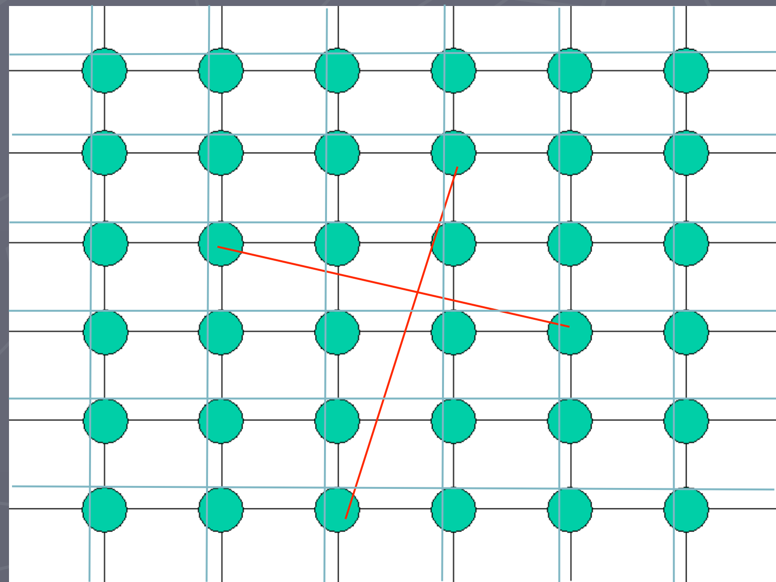
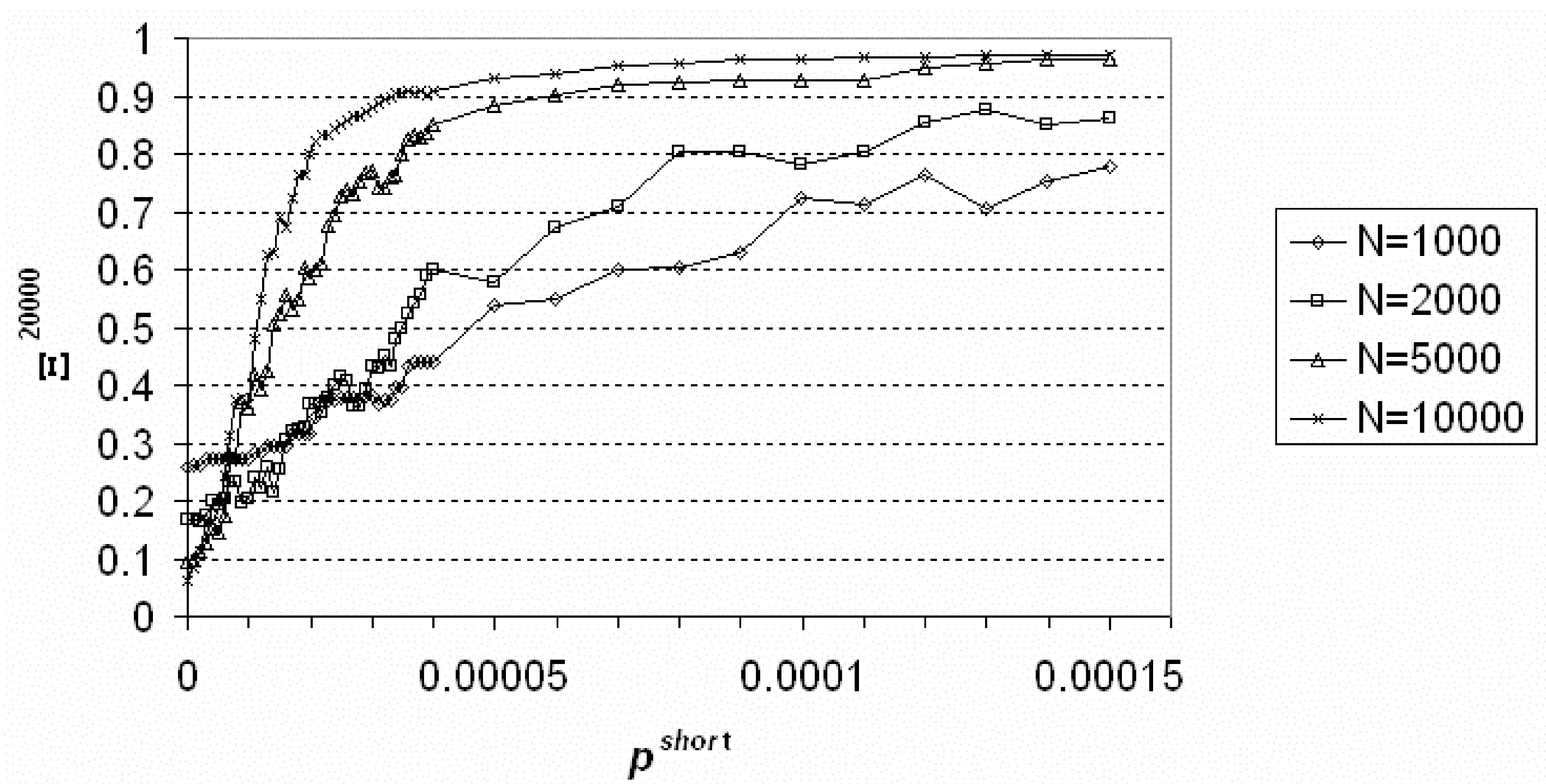




# Discrete Choices on Networks #3

Average values after 20000 iterations for 10x10 runs



# The Proposed Model:
# A possible way of reconciliation

# Context-Preservation Revisited: The Case of Individual Learning

_ Memory length=1, 4 strategies studied:
- Agents have individual probabilities for all 4 strategies.

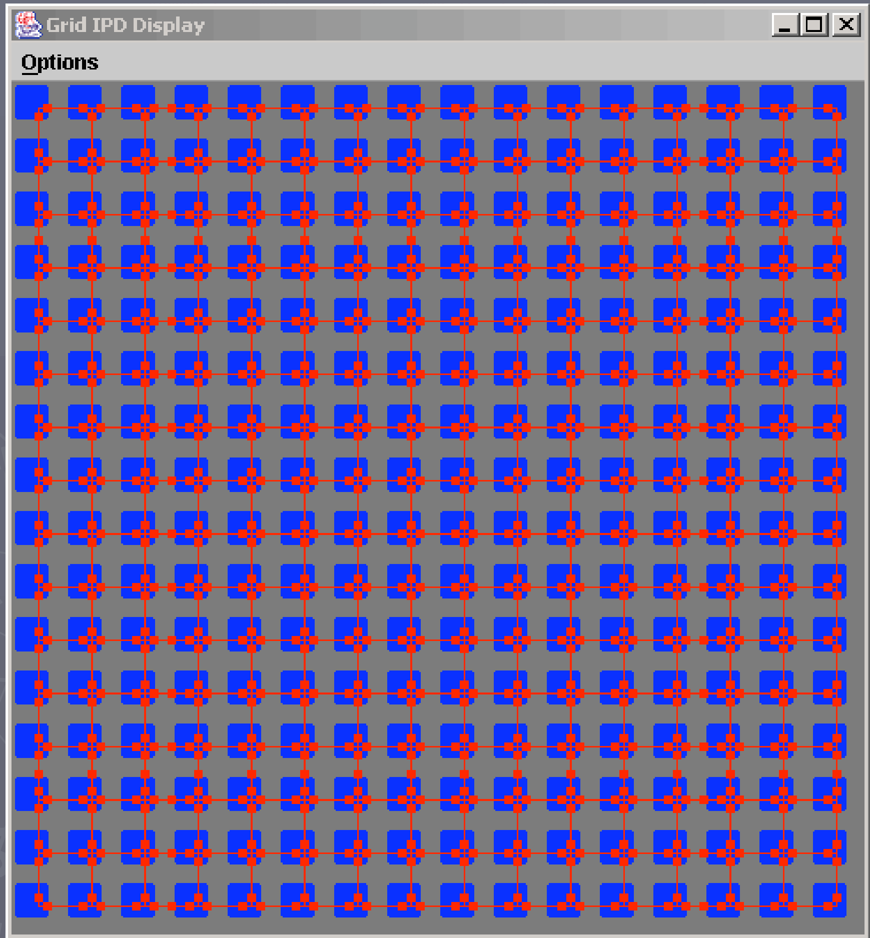
_ Agents are assigned a (uniform) random initial strategy.
  - _ 'ALLC': [**p**, (1-p)/3, (1-p)/3, (1-p)/3]
  - _ 'TFT': [(1-p)/3, **p**, (1-p)/3, (1-p)/3]
  - _ 'ATFT': [(1-p)/3, (1-p)/3, **p**, (1-p)/3]
  - _ 'ALLD': [(1-p)/3, (1-p)/3, (1-p)/3, **p**]

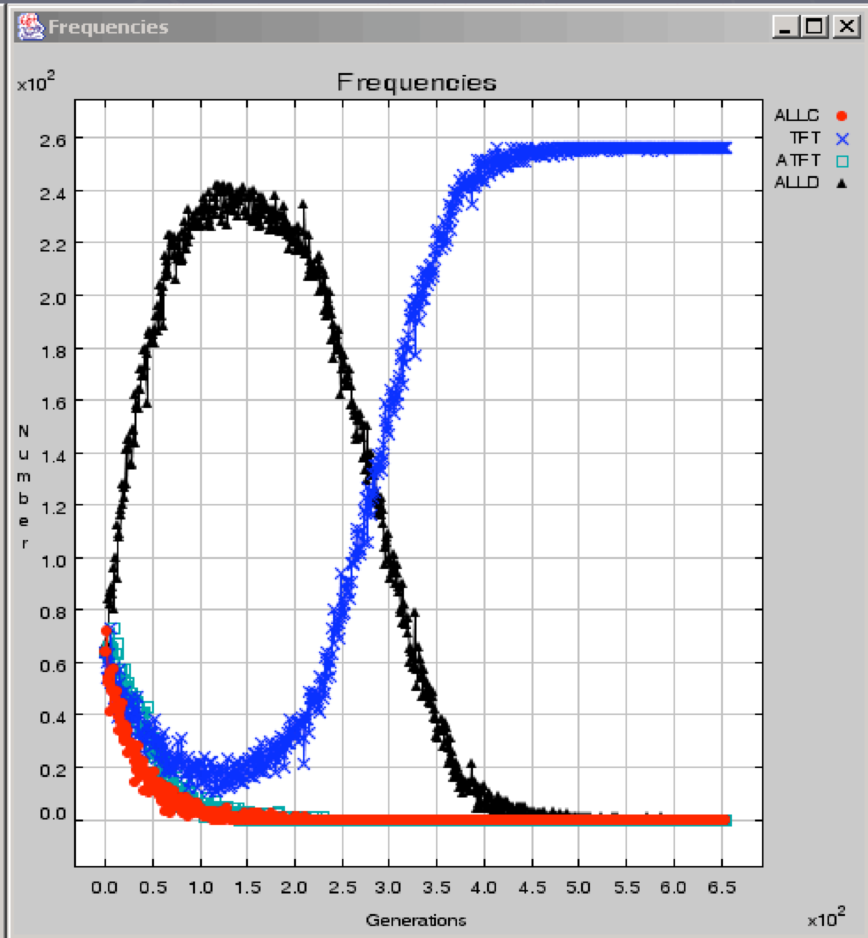
# Context-Preservation Revisited: The Case of Individual Learning #2

- We study Watts-Strogatz networks only
  - Each agent picks a strategy probabilistically for the round.
  - It plays a 4-shots IPD game with each of its neighbors.

- Individual adaptation:
  - At the end of the round, agents increase the probability of the strategy of their most successful neighbors.
    - Probabilities are normalized.
    - Note the convergence properties of the approach.

# Context-Preservation Revisited: The Case of Individual Learning #3

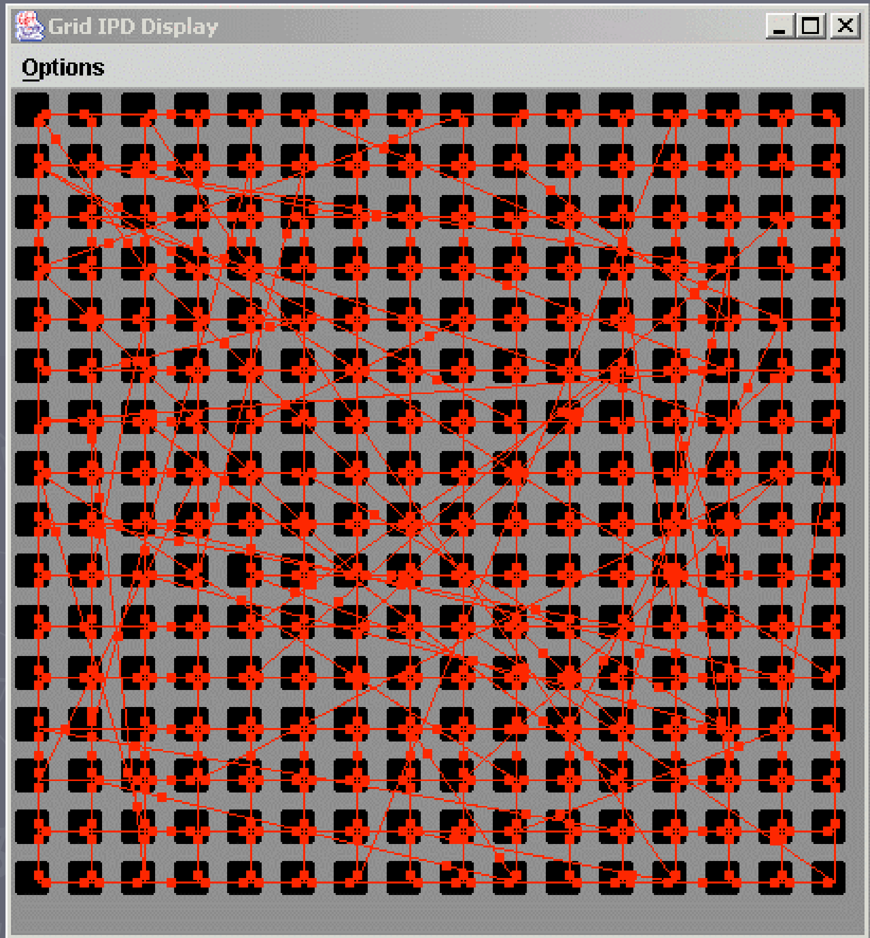


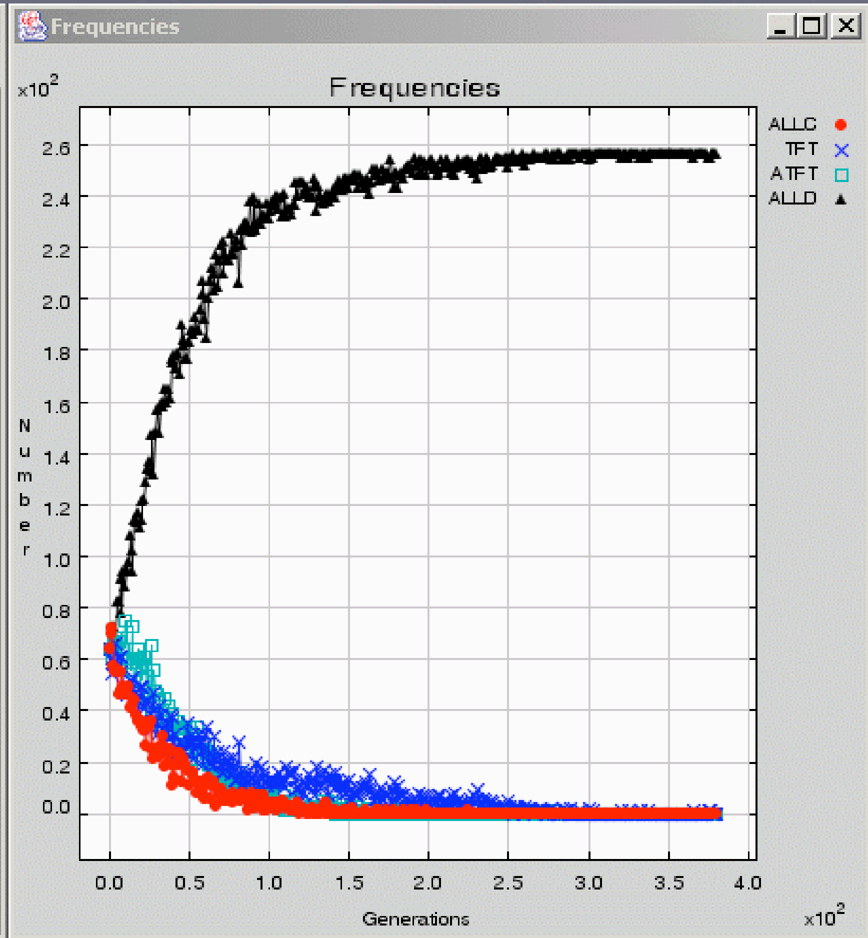






On a 2-dimensional torus

# Context-Preservation Revisited: The Case of Individual Learning #4

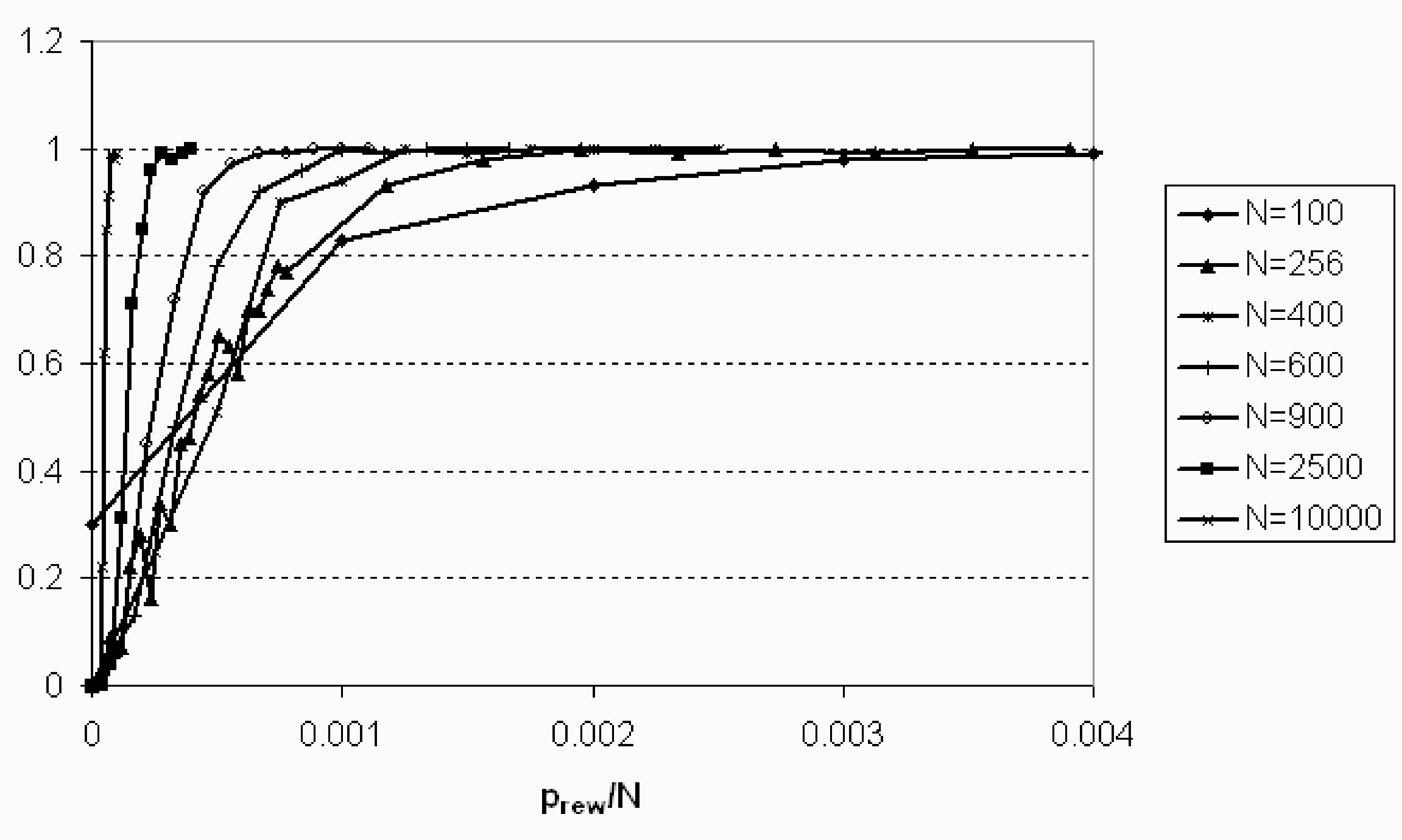





On a 2-dimensional WS-graph

# Cooperation *is* Sensitive to Network Structure




Average number of ALLD choices after 1000 iterations for 10x10 networks

# Cooperation *is* Sensitive to Network Structure Pt. 2

- This sensitivity is *not* dependent on initial configuration.
  - Only to very minor extents.

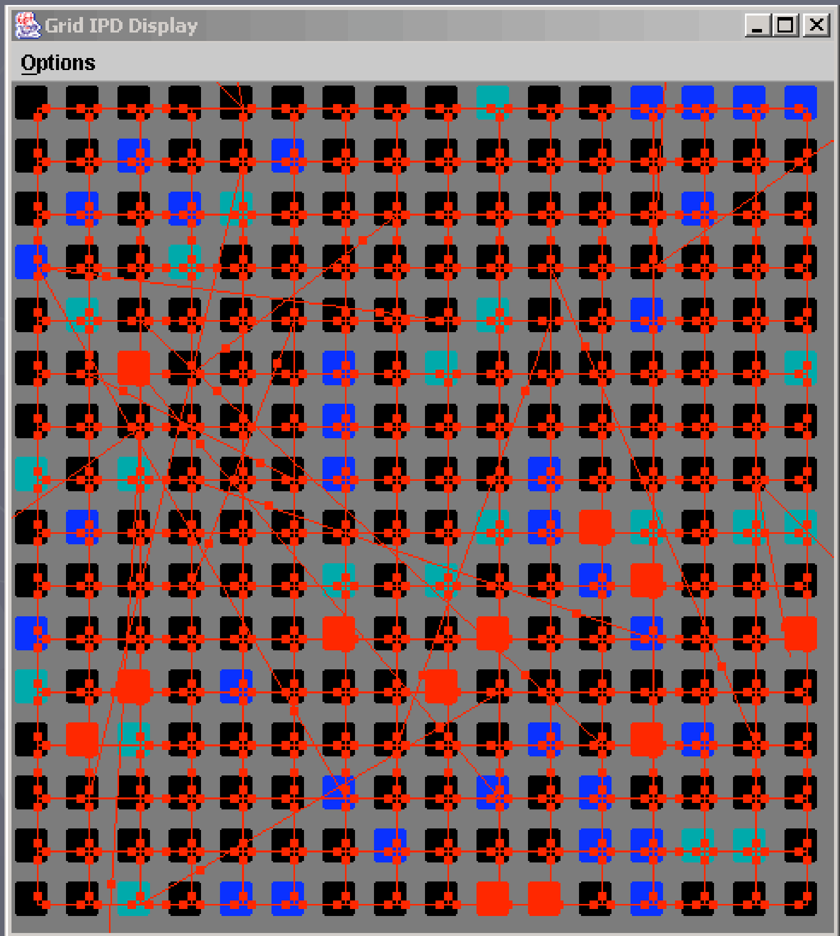
- However, it *is* systematically dependent on the average path length of the underlying network.

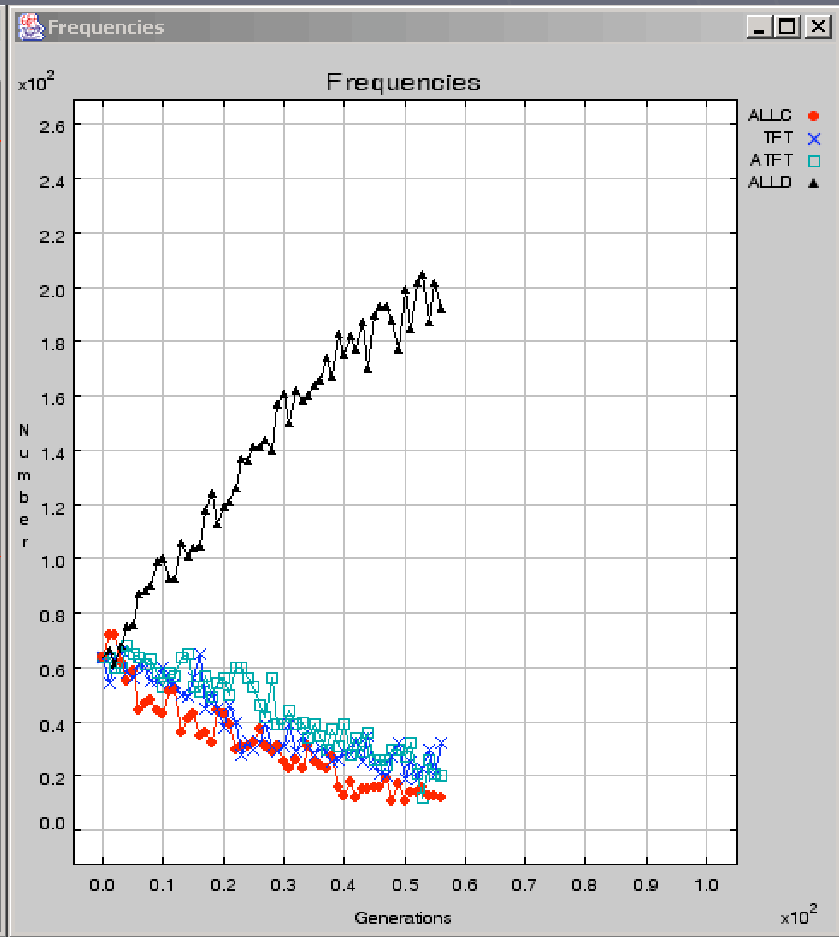
- These results are independent of the value of the initialization parameter **p**.

# A Speculative Explanation (Needs to be confirmed!!)

- Perhaps, the discrete, threshold-like nature of the original model's adaptation rule hinders network-dependence.
  - There are threshold-like 'spreading models on networks' (e.g., by Watts), but those are 'tipping models' (i.e., do not have the option to 'turn back').
- In our model, rare, 'accidental' success of one strategy does not imply an immediate tipping of neighbors.

# Illustration: Individuals do turn back (Progress is less 'smooth')



# Summary
# (Not Quite Conclusions!!)

- Cooperation, as in the IPD game.
- Apparent contradiction about the role of network structure
  - Context-preservation in cooperation games.
    - With constant-like average degrees.
  - Network-sensitivity in discrete choice dynamics.
- A modified model proposed, based on individual adaptation that bridges the gap.